HARRISBURG
CHRISTIAN SCHOOL

Summer Math

for students entering

Pre - Calculus

Name ______________________ Class ______________ Date ______________

Reteaching 1-4

Solving Inequalities

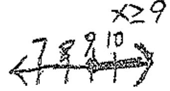
OBJECTIVE: Solving and graphing inequalities **MATERIALS:** None

To solve an inequality, use the techniques used to solve an equation with one difference: when multiplying or dividing each side by a negative number, reverse the inequality.

A $2x-5 \geq 13$ B

$2x \geq 18$

$x \geq 9$



Examples

Solve each inequality. Graph the solutions.

a. $2x - 5 \geq 13$ b. $4 + 3(1 - 2x) > 37$

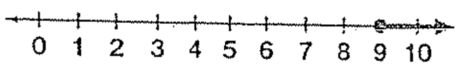
Use the properties of real numbers and the properties of inequalities to rewrite each inequality in equivalent forms.

a. When dividing each side by a positive number, do not reverse the inequality.

$2x - 5 \geq 13$

$2x \geq 18$ ⟵ **Add 5 to each side.**

$x \geq 9$ ⟵ **Divide each side by 2.**



b. When dividing each side by a negative number, reverse the inequality.

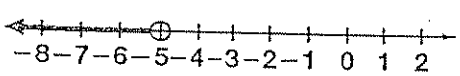
$4 + 3(1 - 2x) > 37$

$4 + 3 - 6x > 37$ ⟵ **Distributive Property**

$7 - 6x > 37$ ⟵ **Simplify.**

$-6x > 30$ ⟵ **Subtract 7 from each side.**

$x < -5$ ⟵ **Divide each side by –6 and reverse the inequality.**



Exercises

Solve each inequality. Graph the solutions.

1. $3(y - 5) \leq 6$
2. $-4t > 2$
3. $3 - 4m < 11$
4. $7d \leq 2(d + 5)$
5. $-2(3 - h) + 2h \geq 0$
6. $3k - (1 - 2k) > 1$
7. $5p + 12 \leq 9p - 20$
8. $3 - 2r < 7 - r$

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◆ Skill B Solving and graphing compound linear inequalities in one variable

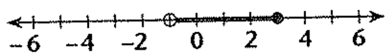
Recall An inequality involving *and* is true only if both parts of the inequality are true.

◆ Example 1

Solve $2x + 3 > 1$ *and* $5x - 9 \le 6$. Graph the solution on a number line.

◆ Solution

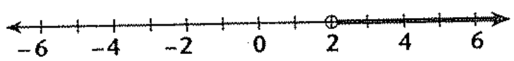
$2x + 3 > 1$		$5x - 9 \le 6$
$2x > -2$		$5x \le 15$
$x > -1$	*and*	$x \le 3$



◆ Example 2

Graph the solution for $x > -1$ *and* $x > 2$.

◆ Solution



Both inequalities are true only if $x > 2$.

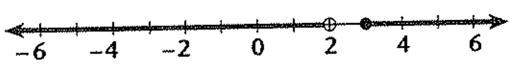
Recall An inequality involving *or* is true if at least one part of the inequality is true.

◆ Example 3

Solve and graph the solution for $2x + 7 \ge 13$ *or* $5x - 4 < 6$.

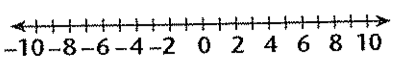
◆ Solution

$2x + 7 \ge 13$		$5x - 4 < 6$
$2x \ge 6$		$5x < 10$
$x \ge 3$	*or*	$x < 2$

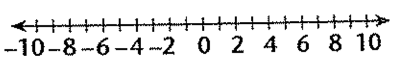


Graph the solution of each compound inequality on a number line.

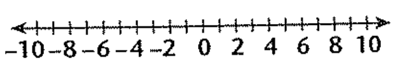
10. $3x + 2 \ge -1$ *and* $3x + 2 \le 8$ __________



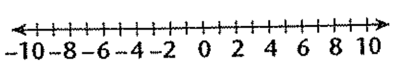
12. $y < -4$ *and* $y > 5$ __________



14. $2z - 1 \le 5$ *or* $3z - 5 > 10$ __________



16. $x < -2$ *or* $x \le 3$ __________



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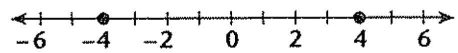
Reteaching
1.8 Solving Absolute-Value Equations and Inequalities

◆ Skill A Solving absolute-value equations and graphing solutions on a number line

Recall The absolute value of x is the distance between x and 0 on the number line.

◆ Example 1
Solve $|x| = 4$. Graph the solution on a number line.

◆ Solution
$x = 4$ *or* $x = -4$



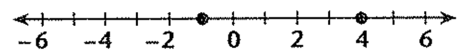
◆ Example 2
Solve $|2x - 3| = 5$. Graph the solution on a number line.

◆ Solution

$2x - 3 = 5$ *or* $2x - 3 = -5$

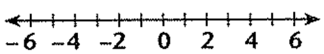
$2x = 8$ $\quad$ $2x = -2$

$x = 4$ *or* $x = -1$

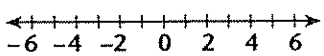


Solve each equation. Graph the solution on a number line.

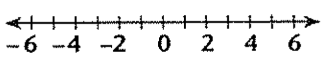
1. $|x| = 3$ ______________________



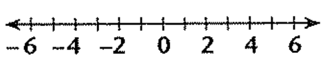
3. $|b - 3| = 3$ ______________________



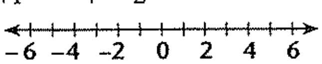
5. $|3 + y| = 2$ ______________________



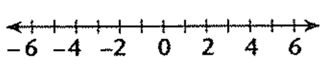
7. $|2a - 5| = 0$ ______________________



9. $\left|\frac{1}{4}t + 1\right| = \frac{1}{2}$ ______________________



11. $|1 - m| - 3 = 1$ ______________________



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◆ Skill B Solving absolute-value inequalities and graphing solutions on a number line

Recall The solution of $|x| < a$, where a is nonnegative, is all real numbers less than a and greater than $-a$.

◆ **Example 1**

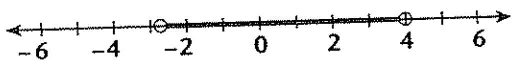
Solve $|3x - 2| < 10$. Graph the solution on a number line.

◆ **Solution**

$3x - 2 < 10$ *and* $3x - 2 > -10$

$3x < 12$ $\qquad$ $3x > -8$

$x < 4$ *and* $x > -2\frac{2}{3}$



Recall The solution of $|x| > a$, where a is nonnegative, is all real numbers less than $-a$ or greater than a.

◆ **Example 2**

Solve $|3 - 2a| - 4 \geq 1$. Graph the solution on a number line.

◆ **Solution**

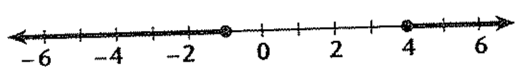
$|3 - 2a| - 4 \geq 1$

$|3 - 2a| \geq 5$

$3 - 2a \geq 5$ *or* $3 - 2a \leq -5$

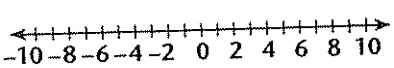
$-2a \geq 2$ $\qquad$ $-2a \leq -8$

$a \leq -1$ *or* $a \geq 4$

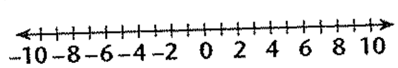


Solve each inequality. Graph the solution on the number line.

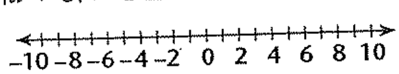
13. $|x| \geq 3$ ______________________



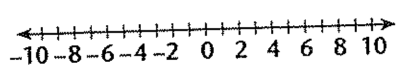
14. $|x| < 3$ ______________________



15. $|a + 3| > 1$ ______________________



16. $|a + 3| \leq 1$ ______________________



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◆ Skill B Simplifying algebraic expressions involving exponents

Recall When you simplify an algebraic expression involving exponents, use Properties of Exponents.

◆ **Example 1**

Simplify $\left(\frac{-x^5}{2x^{-3}y^4}\right)^4$. Write your answer using positive exponents only.

◆ **Solution**

$$\left(\frac{-x^5}{2x^{-3}y^4}\right)^4 = \frac{(-x^5)^4}{(2x^{-3}y^4)^4} \qquad \textit{Power of a Quotient: } \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$= \frac{x^{20}}{16x^{-12}y^{16}} \qquad \textit{Power of a Power: } (a^m)^n = a^{mn}$$

$$= \frac{x^{20-(-12)}}{16y^{16}} \qquad \textit{Quotient of Powers: } \frac{a^m}{a^n} = a^{m-n}$$

$$= \frac{x^{32}}{16y^{16}}$$

◆ **Example 2**

Simplify $5x\left(yx^{-\frac{3}{2}}\right)^{-2}$. Write your answer using positive exponents only.

◆ **Solution**

$$5x\left(yx^{-\frac{3}{2}}\right)^{-2} = 5x(y^{-2}x^3) \qquad \textit{Power of a Product: } (ab)^n = a^nb^n$$

$$= 5x^4y^{-2} \qquad \textit{Power of Powers: } (a^m)^n = a^{mn}$$

$$= \frac{5x^4}{y^2} \qquad \textit{Definition of negative exponent: } a^{-n} = \frac{1}{a^n}$$

Simplify each expression, assuming that no variable equals zero. Write your answer using positive exponents only.

29. $a^2a^4a^3$ ________

30. $x^3(5x)^2$ ________

31. $(3x^2y)(xy^2)$ ________

32. $(a^2b)(-3ab^3)(2ab)$ ________

33. $(c^2d)^3(cd^3)^2$ ________

34. $(2x^2y^3)^3(3x^3y^0)^2$ ________

35. $\left(-\frac{z^3}{y^2}\right)^5$ ________

36. $\left(\frac{4x^2y^{-3}}{y^{-2}}\right)^{-1}$ ________

37. $(x^{-5}y^{-1})^{-2}(x^2y^{-4})^3$ ________

38. $\left(\frac{3x}{y^{-3}}\right)^3\left(\frac{5x^{-10}yz^2}{2x^{-1}y^3}\right)^{-2}$ ________

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Reteaching

2.4 Operations With Functions

◆ **Skill A** Using the four basic operations on functions to write new functions

Recall To write the sum, difference, product, or quotient of two functions, f and g, write the sum, difference, product, or quotient of the expressions that define f and g. Then simplify.

◆ **Example**

Let $f(x) = x^2 + 3x + 2$ and $g(x) = 5x - 1$. Write an expression for each function.

a. $(f + g)(x)$ **b.** $(f - g)(x)$ **c.** $(fg)(x)$ **d.** $\left(\frac{f}{g}\right)(x)$

◆ **Solution**

a. $(f + g)(x) = f(x) + g(x)$

$= (x^2 + 3x + 2) + (5x - 1)$

$= x^2 + 8x + 1$ *Combine like terms.*

b. $(f - g)(x) = f(x) - g(x)$

$= (x^2 + 3x + 2) - (5x - 1)$

$= x^2 + 3x + 2 - 5x + 1$

$= x^2 - 2x + 3$ *Combine like terms.*

c. $(fg)(x) = f(x) \cdot g(x)$

$= (x^2 + 3x + 2)(5x - 1)$

$= (x^2 + 3x + 2)(5x) + (x^2 + 3x + 2)(-1)$ *Distributive Property*

$= 5x^3 + 15x^2 + 10x - x^2 - 3x - 2$

$= 5x^3 + 14x^2 + 7x - 2$

d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$

$= \frac{x^2 + 3x + 2}{5x - 1}$, where $x \neq \frac{1}{5}$

Let $f(x) = 3x^2 + 2$, $g(x) = 2x - 1$, and $h(x) = x^2 + 5x$. Find each new function, and state any domain restrictions.

1. $(f + g)(x)$ ______________________

2. $(f - h)(x)$ ______________________

3. $(h - g)(x)$ ______________________

4. $(gh)(x)$ ______________________

5. $(hg)(x)$ ______________________

6. $(f + h)(x)$ ______________________

7. $\left(\frac{f}{g}\right)(x)$ ______________________

8. $\left(\frac{h}{g}\right)(x)$ ______________________

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◆ Skill B Finding the composite of two functions

Recall To write an expression for the composite function $(f \circ g)(x)$, replace each x in the expression for f with the expression defining g. Then simplify the result.

◆ **Example**

Let $f(x) = 5x$ and $g(x) = 2x^2 - 3$. Find $(f \circ g)(2)$ and $(g \circ f)(2)$. Then write expressions for $(f \circ g)(x)$ and $(g \circ f)(x)$.

◆ **Solution**

$(f \circ g)(2)$: $g(2) = 2(2)^2 - 3 = 5$ $f(g(2)) = f(5) = 5(5) = 25$

Thus, $(f \circ g)(2) = 25$.

$(g \circ f)(2)$: $f(2) = 5(2) = 10$ $g(f(2)) = g(10) = 2(10)^2 - 3 = 197$

Thus, $(g \circ f)(2) = 197$.

To write expressions for $(f \circ g)(x)$ and $(g \circ f)(x)$, use the variable x instead of a particular number.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(2x^2 - 3) \\ &= 5(2x^2 - 3) \\ &= 10x^2 - 15\end{aligned} \qquad \begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(5x) \\ &= 2(5x)^2 - 3 \\ &= 50x^2 - 3\end{aligned}$$

Let $f(x) = x^2 - 1$, $g(x) = 3x$, and $h(x) = 5 - x$. Find each composite function.

9. $(f \circ g)(x)$

____

10. $(g \circ f)(x)$

____

11. $(h \circ f)(x)$

____

12. $(h \circ g)(x)$

____

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Reteaching

3.2 Solving Systems by Elimination

◆ **Skill A** Solving a consistent and independent system of equations by elimination

Recall Two lines with different slopes represent a consistent and independent system of equations. Since these lines intersect in one point, there is one solution to the system.

◆ **Example**

Solve $\begin{cases} 3x + 2y = 13 \\ 4x - 3y = 6 \end{cases}$ by using the elimination method.

◆ **Solution**

To eliminate the y terms multiply each side of the first equation by 3.

$3x + 2y = 13 \rightarrow 3(3x + 2y) = 3(13) \rightarrow 9x + 6y = 39$

Then multiply each side of the second equation by 2.

$4x - 3y = 6 \rightarrow 2(4x - 3y) = 2(6) \rightarrow 8x - 6y = 12$

The system that results is shown below.

$$\begin{aligned} 9x + 6y &= 39 \\ \underline{8x - 6y} &\underline{= 12} \\ 17x &= 51 \quad \textit{Addition Property of Equality} \\ x &= 3 \end{aligned}$$

To find y, replace x with 3 in the first equation.

$$\begin{aligned} 3x + 2y &= 13 \\ 3(3) + 2y &= 13 \\ 2y &= 4 \\ y &= 2 \end{aligned}$$

Check the ordered pair (3, 2) in the second original equation.

$$\begin{aligned} 4x - 3y &= 6 \\ 4(3) - 3(2) &\stackrel{?}{=} 6 \\ 12 - 6 &= 6 \end{aligned}$$

Use elimination to solve each system of equations. Check your solution.

1. $\begin{cases} x - y = -3 \\ x + y = 5 \end{cases}$ ______________________

2. $\begin{cases} x + 3y = 6 \\ x - y = 2 \end{cases}$ ______________________

3. $\begin{cases} 7x - 3y = 32 \\ 2x + y = 11 \end{cases}$ ______________________

4. $\begin{cases} x + 4 = 8 \\ -2x + 5y = 23 \end{cases}$ ______________________

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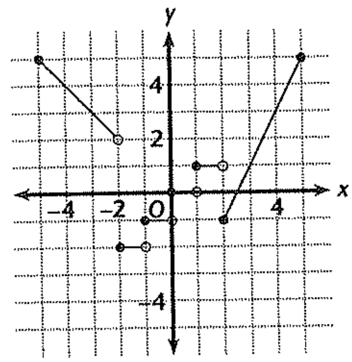
◆ Skill B Graphing piecewise, step, and absolute-value functions

Recall A piecewise function in x is a function defined by different expressions in x on different intervals for x.

◆ Example

Graph this piecewise function.

$$f(x) = \begin{cases} |x|, & \text{if } -5 \le x < -2 \\ [x], & \text{if } -2 \le x < 2 \\ 2x - 5, & \text{if } 2 \le x \le 5 \end{cases}$$



◆ Solution

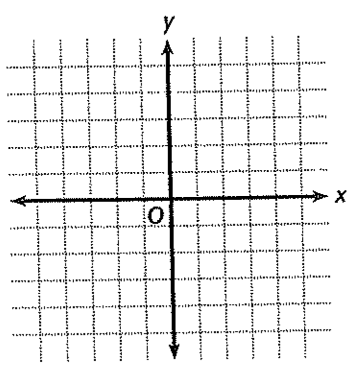
x	-5	-4	-3	-2.5
$y = \|x\|$	5	4	3	2.5

x	-2	-1.5	-1	-0.5	0	1
$y = [x]$	-2	-2	-1	-1	0	1

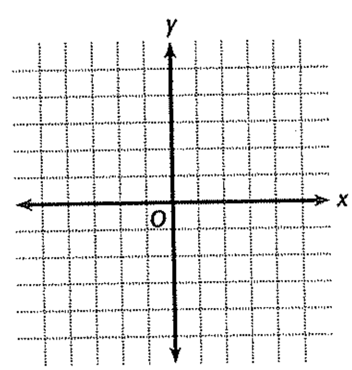
x	2	2.5	3	4	5
$y = 2x - 5$	2	0	1	3	5

Graph each function.

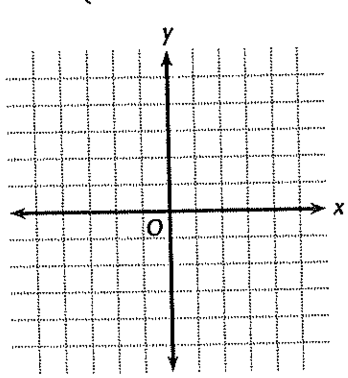
15. $f(x) = \begin{cases} x + 3, \text{ if } x < 0 \\ -2x + 5 \text{ if } x \ge 0 \end{cases}$



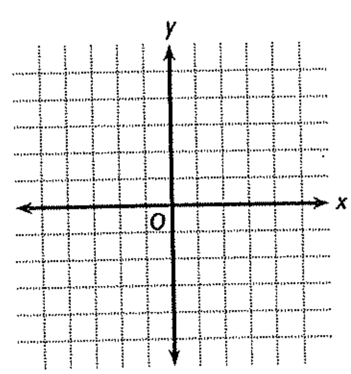
16. $f(x) = \begin{cases} \frac{1}{2}x \text{ if } -4 \le x \le 2 \\ 2x - 3 \text{ if } x > 2 \end{cases}$



17. $f(x) = \begin{cases} |x| \text{ if } x \le 1 \\ 2 - |x - 2| \text{ if } x > 1 \end{cases}$



18. $f(x) = \begin{cases} [x] \text{ if } -2 \le x \le 1 \\ \lceil x \rceil \text{ if } 1 < x \le 4 \end{cases}$



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Reteaching
5.3 Factoring Quadratic Expressions

◆ **Skill A** Factoring a quadratic expression

Recall To factor a quadratic expression, start by looking for the following patterns.

(1) common monomial factor $4a^2 - 20a = 4a(a - 5)$
(2) difference of two squares $4a^2 - 49 = (2a + 7)(2a - 7)$
(3) perfect square trinomial $x^2 - 10x + 25 = (x - 5)^2$

◆ **Example**

Factor each expression.

a. $3x^2 - 27$ **b.** $2x^2 + 16x + 32$ **c.** $x^2 + 5x - 24$

◆ **Solution**

a. $3x^2 - 27 = 3(x^2 - 9)$ *common monomial factor*
$= 3(x + 3)(x - 3)$ *difference of 2 squares*

b. $2x^2 + 16x + 32 = 2(x^2 + 8x + 16)$ *common monomial factor*
$= 2(x + 4)^2$ *perfect square trinomial*

c. None of the basic 3 patterns occurs.
Find the factors of 24. ⟶ $1 \times 24, 2 \times 12, 3 \times 8, 4 \times 6$
Find a pair such that one factor is negative, so that the product is -24, and the sum of the factors is 5.
$-3 \times 8 = -24$ and $-3 + 8 = 5$

So, $x^2 + 5x - 24 = (x - 3)(x + 8)$.

Check by using "FOIL."

Factor each quadratic trinomial.

1. $x^2 + x - 20$ ______________________

2. $2c^2 - 10c - 28$ ______________________

3. $a^2 - 14a + 49$ ______________________

4. $x^2 + 2x + 1$ ______________________

5. $3a^2 + 3a - 6$ ______________________

6. $5c^2 - 80$ ______________________

7. $2z^2 + z - 28$ ______________________

8. $x^2 - 3x - 40$ ______________________

9. $5y^2 - 80y$ ______________________

10. $5r^2 + 23r + 26$ ______________________

11. $9x^2 - 1$ ______________________

12. $d^2 + 10d - 75$ ______________________

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◆ Skill B Using factoring to solve a quadratic equation

Recall If $(x + 3)(x - 5) = 0$, then either $x + 3 = 0$ or $x - 5 = 0$. *Zero Product Property*

◆ Example 1

Solve $2x^2 - 6x - 20 = 0$.

◆ Solution

$2x^2 - 6x - 20 = 0$

$2(x^2 - 3x - 10) = 0$ *common monomial factor*

$2(x - 5)(x + 2) = 0$ *reverse of "FOIL"*

Since $2 \neq 0$,

either $x - 5 = 0$ or $x + 2 = 0$. *Zero Product Property*

$x = 5$ or $x = -2$

The solutions are -2 and 5.

◆ Example 2

Find the zeros of $f(x) = 2x^2 + 7x$.

◆ Solution

$2x^2 + 7x = 0$ *set equal to 0*

$x(2x + 7) = 0$ *common monomial factor*

$x = 0$ or $2x + 7 = 0$ *Zero Product Property*

$x = 0$ or $x = -\frac{7}{2}$

Thus, the zeros are -3.5 and 0.

Solve each quadratic equation by factoring and applying the Zero-Product Property.

13. $x^2 - 7x + 12 = 0$ ______________

14. $3x^2 - 6x = 0$ ______________

15. $x^2 - 8x + 15 = 0$ ______________

16. $x^2 + 5x = 0$ ______________

17. $4x^2 + 9 = -12x$ ______________

18. $a^2 - 2a + 1 = 0$ ______________

19. $12b^2 = -16b$ ______________

20. $x^2 - 49 = 0$ ______________

21. $-8m^2 = 40m$ ______________

Use factoring and the Zero-Product Property to find the zeros of each function.

22. $f(x) = x^2 + 8x + 12$ ______________

23. $f(x) = x^2 + 10x + 25$ ______________

24. $f(x) = 3x^2 - 48$ ______________

25. $f(x) = 7x^2 - 21x + 14$ ______________

26. $f(x) = -5x^2 + 45$ ______________

27. $f(x) = x^2 - 14x - 51$ ______________

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Reteaching

5.6 Quadratic Equations and Complex Numbers

◆ Skill A Using the discriminant to find the number and nature of the roots of a quadratic equation

Recall The discriminant of the equation $ax^2 + bx + c$ is $b^2 - 4ac$.

◆ Example 1

Find the discriminant for each equation. Then tell the number and nature of the solutions.

a. $3x^2 + 12x + 8 = 0$ **b.** $3x^2 + 12x + 12 = 0$ **c.** $3x^2 + 12x + 15 = 0$

◆ Solution

a. $b^2 - 4ac = 12^2 - 4(3)(8) = 48$
Since $48 > 0$, this equation has 2 real solutions.

b. $b^2 - 4ac = 12^2 - 4(3)(12) = 0$
Since the discriminant equals 0, this equation has 1 real solution.

c. $b^2 - 4ac = 12^2 - 4(3)(15) = -36$
Since $-36 < 0$, this equation has 2 nonreal solutions.

Recall $\sqrt{-1} = i$ and $\sqrt{-r} = i\sqrt{r}$, where $r > 0$

◆ Example 2

Use the quadratic formula to solve $2x^2 - 3x + 5 = 0$.

◆ Solution

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(5)}}{2(2)} \longrightarrow x = \frac{3 \pm \sqrt{-31}}{4}$$

$$x = \frac{3}{4} + i\frac{\sqrt{31}}{4} \text{ or } x = \frac{3}{4} - i\frac{\sqrt{31}}{4} \quad \text{where } \sqrt{-31} = i\sqrt{31}$$

Find the discriminant, and determine the number of real solutions.

1. $x^2 + 6x + 3 = 0$ ____________

2. $x^2 + 6x + 10 = 0$ ____________

3. $x^2 - 5x - 5 = 0$ ____________

4. $x^2 + 8x + 16 = 0$ ____________

5. $3x^2 - 4x + 2 = 0$ ____________

6. $x^2 - 4x - 5 = 0$ ____________

Use the quadratic formula to solve each equation.

7. $x^2 - 10x + 34 = 0$ ____________

8. $x^2 - 12x + 32 = 0$ ____________

9. $6x^2 + 7x = -3$ ____________

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◆ Skill B Performing operations with complex numbers

Recall Any power of i can be simplified to i, -1, $-i$, or 1.

$i^2 = -1, i^3 = -i, i^4 = 1, i^5 = i, i^6 = -1, i^7 = -1, i^8 = 1, \ldots$

◆ **Example 1**

Find each sum, difference, product, and quotient.

a. $(2 + 3i) + (4 - i)$

b. $(2 + 3i) - (4 - i)$

c. $(2 + 3i)(4 - i)$

d. $(2 + 3i) \div (4 - i)$

◆ **Solution**

a. $(2 + 3i) + (4 - i)$
$= (2 + 4) + (3i - i)$
$= 6 + 2i$

b. $(2 + 3i) - (4 - i)$
$= (2 - 4) + (3i - (-i))$
$= -2 + 4i$

c. $(2 + 3i)(4 - i)$
$= 8 - 2i + 12i - 3i^2$
$= -2i + 12i - 3(-1)$
$= (8 + 3) + (-2 + 12)i$
$= 11 + 10i$

d. $(2 + 3i) \div (4 - i)$

$= \frac{2 + 3i}{1 - 4i} \cdot \frac{1 + 4i}{1 + 4i}$

$= \frac{2 + 8i + 3i + 12i^2}{-15}$

$= \frac{-10 + 11i}{-15} = \frac{2}{3} - \frac{11}{15}i$

Recall The absolute value of $a + bi$, denoted $|a + bi|$, is given by $\sqrt{a^2 + b^2}$.

◆ **Example 2**

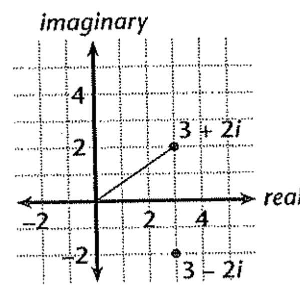
Graph $3 + 2i$ and its conjugate on the complex plane. Then find the absolute value of $2 + 3i$.

◆ **Solution**

The graphs of $3 + 2i$ and $3 - 2i$ are shown at right.

$|3 + 2i| = \sqrt{3^2 + 2^2} = \sqrt{13}$

The length of the segment from the origin to the point representing $3 + 2i$ is $\sqrt{13}$.



Perform the indicated operation.

10. $(-3 + i) + (6 - 3i)$ ______________

11. $(-8 + 5i) + (-8 - 5i)$ ______________

12. $(2 - i) - (3 + 2i)$ ______________

13. $(-7 + 2i) - (4 - 5i)$ ______________

14. $(3 + i)(-4 + 2i)$ ______________

15. $(5 + 2i)(5 - 2i)$ ______________

16. $(5 + i) \div (2 - i)$ ______________

17. $(3 - 4i) \div (1 + i)$ ______________

Write the conjugate and find the absolute value of each complex number.

18. $8 - 5i$ ______________

19. $-2 + i$ ______________

20. $-5 - 5i$ ______________

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Reteaching
6.3 Logarithmic Functions

◆ Skill A Writing equations in equivalent exponential or logarithmic form

Recall If b is positive and $b \neq 1$, then $x = \log_b y$ if and only if $b^x = y$.

◆ Example 1
Write each equation in exponential form.

a. $\log_5 25 = 2$ **b.** $\log_{10} 1000 = 3$ **c.** $\log_3 \frac{1}{9} = -2$

◆ Solution

a. $\log_5 25 = 2$ indicates that you must raise the base 5 to the power 2 to get 25.
$5^2 = 25$

b. $\log_{10} 1000 = 3$ is equivalent to $10^3 = 1000$.

c. $\log_3 \frac{1}{9} = -2$ is equivalent to $3^{-2} = \frac{1}{9}$. *Recall that* $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

◆ Example 2
Write each equation in logarithmic form.

a. $2^5 = 32$ **b.** $4^{-2} = \frac{1}{16}$ **c.** $8^{\frac{1}{3}} = 2$

◆ Solution

a. $2^5 = 32$ is equivalent to $\log_2 32 = 5$

b. $4^{-2} = \frac{1}{16} \Rightarrow \log_4 \frac{1}{16} = -2$

c. $8^{\frac{1}{3}} = \Rightarrow \log_8 2 = \frac{1}{3}$

Write each equation in logarithmic form.

1. $8^2 = 64$ ______________

2. $3^3 = 27$ ______________

3. $5^4 = 625$ ______________

4. $16^{\frac{1}{2}} = 4$ ______________

5. $5^{-2} = \frac{1}{25}$ ______________

6. $\left(\frac{1}{64}\right)^{\frac{1}{2}} = \frac{1}{8}$ ______________

Write each equation in exponential form.

7. $\log_7 49 = 2$ ______________

8. $\log_{10} 10{,}000 = 4$ ______________

9. $\log_2 64 = 6$ ______________

10. $\log_2 \frac{1}{8} = -3$ ______________

11. $\log_8 8 = 1$ ______________

12. $\log_5 1 = 0$ ______________

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◆ **Skill B** Solving equations using the definitions of exponential and logarithmic functions

Recall The One-to-One Property of Exponential Functions states that if $b^x = b^y$, then $x = y$.

◆ **Example**

Find the value of n in each equation.

a. $n = \log_5 125$ **b.** $4 = \log_2 n$ **c.** $\log_n 100{,}000 = 5$

◆ **Solution**

a. $n = \log_5 125 \rightarrow 5^n = 125$
$5^n = 5^3$
$n = 3$

b. $4 = \log_2 n \rightarrow 2^4 = n$
$16 = n$

c. $\log_n 100{,}000 = 5 \rightarrow n^5 = 100{,}000$
$n^5 = 10^5$
$n = 10$

Find the value of *n* in each equation.

13. $n = \log_{10} 100$ ______

14. $n = \log_2 16$ ______

15. $n = \log_8 8$ ______

16. $\log_9 n = 2$ ______

17. $\log_2 n = 5$ ______

18. $\log_4 n = 0$ ______

19. $\log_n 125 = 3$ ______

20. $\log_n 1{,}000{,}000 = 6$ ______

21. $\log_n \frac{1}{8} = 3$ ______

22. $n = \log_7 1$ ______

23. $\log_n 243 = 5$ ______

24. $\log_{\frac{1}{4}} n = 3$ ______

25. $\log_n \frac{1}{49} = -2$ ______

26. $n = \log_{\frac{1}{2}} 16$ ______

27. $\log_5 n = 1$ ______

28. $n = \log_{\frac{1}{5}} 125$ ______

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Reteaching
6.4 Properties of Logarithmic Functions

◆ **Skill A** Using the properties of logarithms to write equivalent logarithmic expressions

Recall Three properties of logarithms are:

$\log_b(mn) = \log_b m + \log_b n$ $\quad \log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$ $\quad \log_b m^p = p \log_b m$

◆ **Example 1**

Write each expression as a single logarithm.

a. $\log_3 9 + \log_3 3$ **b.** $\log_5 625 - \log_5 125$ **c.** $3 \log_2 r + 5 \log_2 t$

◆ **Solution**

a. $\log_3 9 + \log_3 3 = \log_3(9 \cdot 3)$ *Product Property of Logarithms*
$= \log_3 27$

b. $\log_5 625 - \log_5 125 = \log_5\left(\frac{625}{125}\right)$ *Quotient Property of Logarithms*
$= \log_5 5$

c. $3 \log_2 r + 5 \log_2 t = \log_2 r^3 + \log_2 t^5$ *Power Property of Logarithms*
$= \log_2 r^3t^5$ *Product Property of Logarithms*

◆ **Example 2**

Given $\log_{10} 2 \approx 0.3010$ and $\log_{10} 3 \approx 0.4771$, find each logarithm.

a. $\log_{10} 6$ **b.** $\log_{10} 15$ **c.** $\log_{10} 18$

◆ **Solution**

a. $\log_{10} 6 = \log_{10}(2 \cdot 3)$
$= \log_{10} 2 + \log_{10} 3$
$\approx 0.3010 + 0.4771$
≈ 0.7781

b. $\log_{10} 1.5 = \log_{10}\left(\frac{3}{2}\right)$
$= \log_{10} 3 - \log_{10} 2$
$\approx 0.4771 - 0.3010$
≈ 0.1761

c. $\log_{10} 18 = \log(2 \cdot 3^2)$
$= \log_{10} 2 + 2 \log_{10} 3$
$\approx 0.3010 + 2(0.4771)$
≈ 1.2552

Write each expression as a single logarithm. Then simplify if possible.

1. $\log_2 5 + \log_2 8$ ____________

2. $\log_5 20 - \log_5 2$ ____________

3. $\log_8 16 + \log_8 10$ ____________

4. $\log_3 15 - \log_3 5$ ____________

5. $2 \log_3 2 + 3 \log_3 4$ ____________

6. $5 \log_6 x - 3 \log_6 y$ ____________

Use $\log_{10} 3 \approx 0.4771$ and $\log_{10} 5 \approx 0.6990$ to evaluate each logarithm without using a calculator.

7. $\log_{10} 15 \approx$ ____________

8. $\log_{10} 0.6 \approx$ ____________

9. $\log_{10} 25 \approx$ ____________

10. $\log_{10} 27 \approx$ ____________

11. $\log_{10} 75 \approx$ ____________

12. $\log_{10} 135 \approx$ ____________

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◆ Skill B Using the inverse property of exponents and logarithms

Recall Since $f(x) = b^x$ and $g(x) = \log_b x$ are inverse functions, $\log_b b^x = x$ and $b^{\log_b x} = x$ for $x > 0$.

◆ Example

Simplify each expression.

a. $\log_5 5^4$ **b.** $3^{\log_3 9}$

◆ Solution

a. $\log_5 5^4 = 4$

Notice in this example that

$\log_5 5^4 = \log_5 625$

$= 4$

b. $3^{\log_3 9} = 9$

Notice in this example that

$3^{\log_3 9} = 3^2$

$= 9$

Using the inverse property of exponents and logarithms, simplify each expression.

13. $\log_6 6^3$ ________ **14.** $5^{\log_5 13}$ ________ **15.** $12^{\log_{12} 5}$ ________ **16.** $\log_{10} 10^8$ ________

◆ Skill C Solving an equation involving logarithms.

Recall The One-to-One Property of Logarithmic Functions states that if $\log_b x = \log_b y$, then $x = y$.

◆ Example

Solve $\log_5(x^2 - 10) = \log_5 3x$ for x, and check your answers.

◆ Solution

$\log_5(x^2 - 10) = \log_5 3x$

$x^2 - 10 = 3x$ *One-to-One Property of Logarithms*

$x^2 - 3x - 10 = 0$

$(x - 5)(x + 2) = 0$

$x = 5$ or $x = -2$

Check:

$\log_5(5^2 - 10)$ $\log_5(3 \cdot 5)$

$\log_5 15 = \log_5 15$

true

$\log_5((-2)^2 - 10)$ $\log_5(3 \cdot (-2))$

$\log_5(-6)$ $\log_5(-6)$

undefined

Since $x = -2$ is undefined and $x = 5$ is true, the solution is $x = 5$.

Solve each equation for x.

17. $\log_3(x^2 + 5x) = \log_3 24$ **18.** $\log_8 x^2 = \log_8 16$ **19.** $\log_2(x^2 + 28) = \log_2 11x$

________ ________ ________

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Reteaching

6.5 Applications of Common Logarithms

◆ **Skill A** Using logarithms to solve exponential equations

Recall The common logarithm, $\log_{10} x$, is usually written as $\log x$.

◆ **Example**

Solve each equation.

a. $3^x = 81$ **b.** $5^x = 75$ **c.** $7^{x+1} = 150$

◆ **Solution**

a. $3^x = 81$

Since 81 is a power of 3, use powers of 3.

$3^x = 3^4$

$x = 4$ *One-to-One Property of Exponential Functions*

b. $5^x = 75$

Since 75 is **not** a power of 5, use logarithms to solve this equation.

$\log 5^x = \log 75$

$x \log 5 = \log 75$ *Power Property of Logarithms*

$x = \frac{\log 75}{\log 5}$

$x \approx 2.68$

Check: $5^{2.68} \approx 75$

c. $7^{x+1} = 150$

$\log 7^{x+1} = \log 150$

$(x + 1)\log 7 = \log 150$

$x + 1 = \frac{\log 150}{\log 7}$

$x = \frac{\log 150}{\log 7} - 1$

$x \approx 1.57$

Solve each equation. Round your answers to the nearest hundredth.

1. $7^x = 80$ ______

2. $5^x = 10$ ______

3. $6^x = 1296$ ______

4. $4^{x+1} = 100$ ______

5. $2^{x-3} = 25$ ______

6. $3^{x+4} = 27$ ______

7. $6^{2x-7} = 216$ ______

8. $5^{3x-1} = 49$ ______

9. $10^{x+5} = 125$ ______

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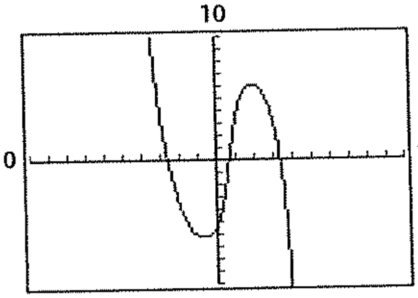
Reteaching
7.2 Polynomial Functions and Their Graphs

◆ **Skill A** Describing the graph of a polynomial function (You will need a graphics calculator.)

Recall Graphs are always read from left to right.
A function is increasing from left to right if the y-values are increasing.
A function is decreasing from left to right if the y-values are decreasing.

◆ **Example**
Given $P(x) = -x^3 + 2x^2 + 5x - 4$:
a. graph the function,
b. give the approximate coordinates of each local minimum and maximum,
c. find the intervals over which the function is increasing, and
d. find the intervals over which the function is decreasing.

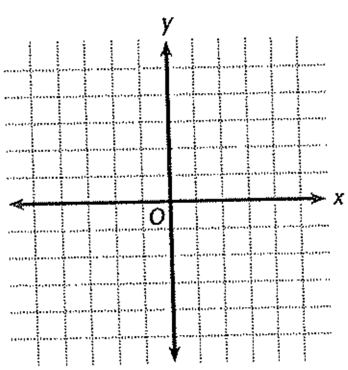
◆ **Solution**
a. The graph is shown at right.



b. Local minimum: approximately $(-0.8, -6.2)$
For all values of x close to -0.8, the y-coordinate is greater than -6.2.
Local maximum: approximately $(2.1, 6.1)$
For all values of x close to 2.1, the y-coordinate is less than 6.1.
c. increasing when $-0.8 < x < 2.1$
d. decreasing when $x < -0.8$ and also when $x > 2.1$

Graph each function. Find any local maxima or minima to the nearest tenth, and the intervals over which the function is increasing and decreasing.

1. $P(x) = x^3 + 4x^2 - 7$



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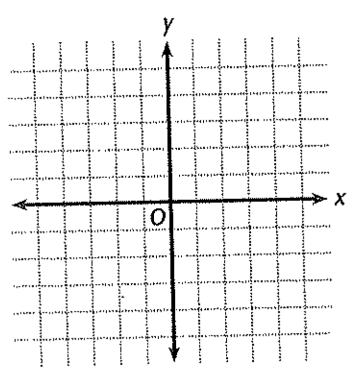
2. $P(x) = x^4 - 3x^2 + 5$



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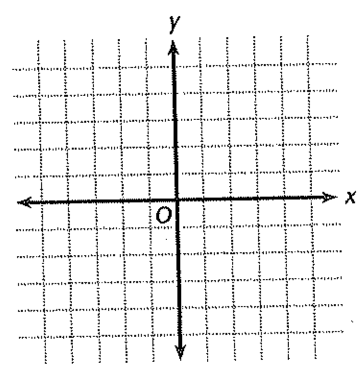
3. $P(x) = -3x^2 + 11x - 2$



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◆ Skill B Describing the end behavior and the number of turning points of the graph of a polynomial function (You will need a graphics calculator.)

Recall The degree of a polynomial is determined by the greatest power of the variable.

◆ Example

Describe the end behavior and state the number of turning points for each graph shown below.

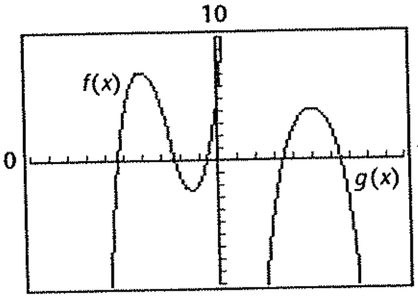
◆ Solution

$f(x)$: As you read across the graph from left to right, this function is rising on the left and on the right. There are 2 turning points in between.

Cubic functions have *at most* 2 turning points.

$g(x)$: As you read across the graph from left to right, this function is rising on the left but falling on the right with one turning point.

Quadratic functions have one turning point.



Describe the end behavior and number of turning points for the graph of each function.

4. $f(x) = x^3 - 7x + 2$ ______

5. $f(x) = -x^3 + 2x^2 - 4$ ______

6. $f(x) = x^2 + 5x - 1$ ______

7. $f(x) = -x^2 + x + 5$ ______

8. $f(x) = x^5 - 5x^3 + 4x$ ______

9. $f(x) = -x^4 + 5x - 1$ ______

Without using a calculator, answer each question. Then check using a graphics calculator.

10. Exercise 4 is an odd-degree function (degree 3) and the coefficient of x^3 is positive (+1). Using this as a pattern, predict the end behavior and greatest possible number of turning points of the graph of the function $f(x) = x^5 - 15x^4 + 85x^3 - 225x^2 + 274x - 120$.

______

11. Exercise 5 is an odd-degree function and the coefficient of x^3 is negative. Using this as a pattern, predict the end behavior and the greatest possible number of turning points of the graph of the function $f(x) = -x^5 + 2x^2 - 1$.

______

12. Using the even-degree function in Exercise 6 as a model, predict the end behavior and greatest possible number of turning points of the graph of the function $f(x) = x^4 - 4x^3 - 2x^2 + 12x + 1$.

______

13. Using the even-degree function in Exercise 7 as a model, predict the end behavior and greatest possible number of turning points of the graph of the function $f(x) = -x^4 + 3x$.

______

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Reteaching
7.4 Solving Polynomial Equations

◆ Skill A Using graphs, synthetic division, and factoring to find rational roots

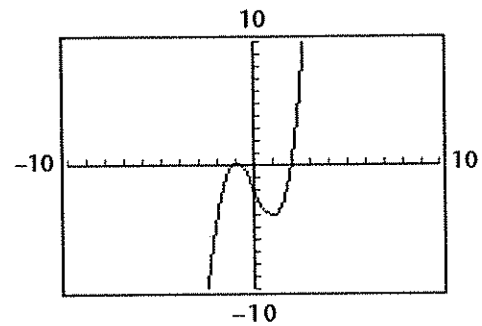
Recall If 3 is a zero of $P(x) = x^2 + x - 12$, then $x = 3$ is a solution (root) of $x^2 + x - 12 = 0$ and $x - 3$ is a factor of $x^2 + x - 12$.

◆ Example

Find the roots of $x^3 - 3x - 2 = 0$.

◆ Solution

Graph the function $f(x) = x^3 - 3x - 2$ and check for zeros of the function. (These occur where the graph crosses the x-axis.) The graph indicates that 2 may be a root of the given equation.



Use synthetic division to check if 2 is a root.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -3 & -2 \\ & & 2 & 4 & 2 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

← *The zero remainder indicates that 2 is a root.*

This means that $x - 2$ is a factor of $x^3 - 3x - 2$.

Thus, $x^3 - 3x - 2 = (x - 2)(x^2 + 2x + 1)$. (You read the coefficients of the second factor, $x^2 + 2x + 1$, from the first three numbers, 1, 2, and 1, found in the last line of the synthetic division above.)

Factor $x^2 + 2x + 1$: $x^2 + 2x + 1 = (x + 1)^2$

Solve $(x - 2)(x + 1)^2 = 0$.

$$(x - 2)(x + 1)^2 = 0$$

$$x - 2 = 0 \text{ or } x + 1 = 0 \text{ or } x + 1 = 0 \qquad \textit{Zero-Product Property}$$

$$x = 2 \qquad x = -1 \qquad x = -1$$

The roots are 2 and -1, with -1 occurring twice.

Use a graph, synthetic division, and factoring to find all of the roots of each equation.

1. $4x^3 + 8x^2 - 11x + 3 = 0$

____

3. $2x^3 + x^2 - 54x + 72 = 0$

____

5. $x^3 - 4x^2 - 28x - 32 = 0$

____

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Reteaching
7.5 Zeros of Polynomial Functions

◆ **Skill A** Finding all the zeros of a polynomial function

Recall A corollary to the Fundamental Theorem of Algebra states that an n^{th}-degree polynomial function will have exactly n complex zeros.

◆ **Example**

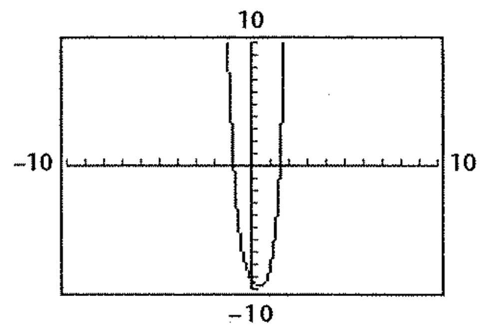
Find all the zeros of $f(x) = \underline{2}x^4 - x^3 + 3x^2 - 3x \underline{-9}$.

◆ **Solution**

The only possible *rational* roots are those found by using a factor of –9 (constant term) as the numerator of a fraction and a factor of 2 (coefficient of x^4) as the denominator.

possibilities: $\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{9}{1}, \pm\frac{9}{2}$

The graph shows that one zero appears to be -1.



$$\begin{array}{r|rrrrr} \underline{-1} & 2 & -1 & 3 & -3 & -9 \\ & & -2 & 3 & -6 & 9 \\ \hline & 2 & -3 & 6 & -9 & 0 \end{array}$$

The zero remainder indicates that -1 is a zero.

The graph shows that there is another zero between 2 and 3. The only possible rational root from the list above between 2 and 3 is $\frac{3}{2}$, or 1.5.

To see if 1.5 is a root, apply synthetic division to $2x^3 - 3x^2 + 6x - 9$ found from the last line of the synthetic division above.

$$\begin{array}{r|rrrr} \underline{1.5} & 2 & -3 & 6 & -9 \\ & & 3 & 0 & 9 \\ \hline & 2 & 0 & 6 & 0 \leftarrow \textit{zero remainder} \end{array}$$

So 1.5 is also a zero.

This leaves a last row (without the remainder) which represents $2x^2 + 6$.

Use the quadratic formula where $a = 1$, $b = 0$, and $c = 6$.

$$x = \frac{-0 \pm \sqrt{0^2 - 4(2)(6)}}{2(2)} = \pm\frac{\sqrt{-48}}{4} = \pm\frac{4i\sqrt{3}}{4} = \pm i\sqrt{3}$$

The four zeros of $f(x) = 2x^4 - x^3 + 3x^2 - 3x - 9$ are -1, 1.5, $i\sqrt{3}$, and $-i\sqrt{3}$.

Find all of the rational roots of each polynomial equation.

1. $f(x) = x^3 + x^2 - x + 15$

____

____

2. $f(x) = x^4 + x^3 - 6x^2 - 14x - 12$

____

____

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Reteaching

8.2 Rational Functions and Their Graphs

◆ **Skill A** Finding the domain of a rational function

Recall Division by zero is not allowed; it is undefined.

◆ **Example**

Find the domain of $g(x) = \frac{x^2 + 8x}{x^2 + 3x - 10}$.

◆ **Solution**

You must exclude from the domain any values of x which cause the denominator to have a value of 0.

Set $x^2 + 3x - 10$ equal to 0.

$(x + 5)(x - 2) = 0$

$x = -5$ or $x = 2$

The domain is all real numbers except -5 and 2.

Find the domain of each rational function.

1. $\frac{x + 3}{x^2 - 16}$ ____________

2. $\frac{5x}{x^2 + 7x}$ ____________

3. $\frac{x^2 + 5}{x^2 - 4x - 21}$ ____________

◆ **Skill B** Identifying vertical asymptotes and holes in the graph of a rational function

Recall If $(x - b)$ is a factor in both the numerator and denominator, there will be a hole in the graph at $x = b$. If $(x - b)$ is a factor of the denominator but not a factor of the numerator, there will be a vertical asymptote of $x = b$.

◆ **Example**

For the rational function $f(x) = \frac{2x^2 + 3x - 2}{x^2 - x - 6}$

a. identify the x-coordinates of any holes in the graph.

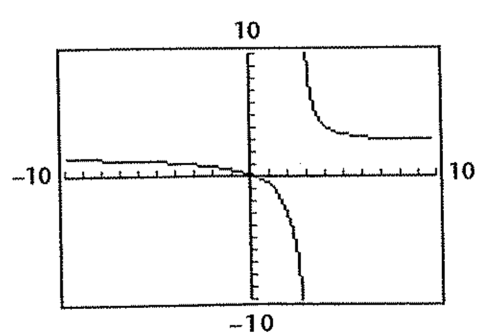
b. write the equations of any vertical asymptotes.

◆ **Solution**

a. $\frac{2x^2 + 3x - 2}{x^2 - x - 6} = \frac{(2x - 1)(x + 2)}{(x + 2)(x - 3)}$

Since $x + 2$ is a factor of both the numerator and denominator, there will be a hole at $x = -2$.

b. Since $x - 3$ is a factor of the denominator but not the numerator, and has a value of 0 when $x = 3$, there will be a vertical asymptote at $x = 3$.



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Identify all holes and asymptotes in the graph of each rational function.

4. $f(x) = \dfrac{(x-3)(x+2)}{(x+3)(x+2)}$ ______________________

5. $f(x) = \dfrac{x+5}{(x-1)(x+4)}$ ______________________

6. $f(x) = \dfrac{x^2-4x+3}{x^2-x-6}$ ______________________

7. $f(x) = \dfrac{x^2+6x-7}{x-1}$ ______________________

◆ **Skill C** Writing an equation for the horizontal asymptote of a graph

Recall The degree of a polynomial is the greatest degree of its terms.

◆ **Example**

Write the equation of the horizontal asymptote for each of the following functions.

a. $f(x) = \dfrac{2x^2+3x-2}{x^2-x-6}$ **b.** $g(x) = \dfrac{3x}{x^2-5}$ **c.** $h(x) = \dfrac{-x^2}{x-2}$

◆ **Solution**

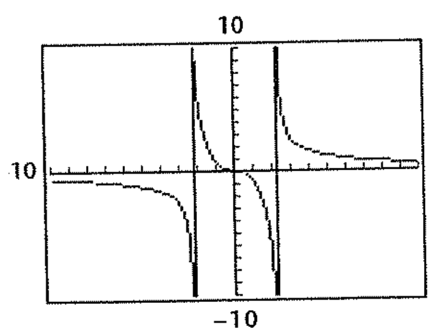
a. Since the degree of the numerator and denominator are the same, divide the coefficient of the term with the greatest degree in the numerator by the coefficient of the like term in the denominator.

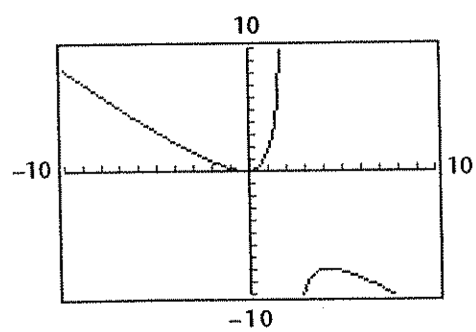
$\frac{2}{1} = 2$ (coefficients of the x^2 terms)

Thus, $y = 2$ is a horizontal asymptote. The graph is shown on the preceding page.

b. Since the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $y = 0$. (The graph is shown at left below.)

c. Since the degree of the numerator is greater than the degree of the denominator, there is no horizontal asymptote. (The graph is shown at right below.)





Identify any horizontal asymptotes for the following functions. Use a graphics calculator to check your answer.

8. $f(x) = \dfrac{5x^2+8}{2x^2-3x}$ ______________

9. $f(x) = \dfrac{x^2+5x-6}{x+2}$ ______________

10. $f(x) = \dfrac{x+5}{(x-1)(x+4)}$ ______________

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8.3 Multiplying and Dividing Rational Expressions

◆ Skill A Simplifying rational expressions

Recall To simplify rational expressions, start by factoring the numerator and denominator.

◆ Example 1

Simplify. $\dfrac{x^2 - 25}{x^2 - 3x - 10}$

◆ Solution

$$\frac{x^2 - 25}{x^2 - 3x - 10} = \frac{(x + 5)(x - 5)}{(x + 5)(x - 2)}$$ *Factor numerator and denominator.*

$$= \frac{x - 5}{x - 2}$$ *Divide numerator and denominator by x + 5.*

◆ Example 2

Simplify $\dfrac{5}{9y^2} \cdot \dfrac{2y^4}{10} \cdot \dfrac{3}{7y}$.

◆ Solution

$$\frac{5}{9y^2} \cdot \frac{2y^4}{10} \cdot \frac{3}{7y} = \frac{\cancel{5} \cdot 2 \cdot \cancel{3} \cdot y^4}{\cancel{3} \cdot 3 \cdot 2 \cdot \cancel{5} \cdot 7 \cdot y^2 \cdot y}$$
$$= \frac{1y^4}{21y^3}$$
$$= \frac{y}{21}$$

◆ Example 3

Simplify $\dfrac{x^2 + 5x + 6}{x^2 - x - 20} \cdot \dfrac{x^2 + 3x - 4}{x^2 + x - 2}$.

◆ Solution

$$\frac{x^2 + 5x + 6}{x^2 - x - 20} \cdot \frac{x^2 + 3x - 4}{x^2 + x - 2} = \frac{\cancel{(x + 2)}(x + 3)\cancel{(x + 4)}\cancel{(x - 1)}}{(x - 5)\cancel{(x + 4)}\cancel{(x + 2)}\cancel{(x - 1)}} = \frac{x + 3}{x - 5}$$

Simplify each rational expression.

1. $\dfrac{3x - 6}{x^2 - 5x + 6}$

____

3. $\dfrac{4x^2}{5} \cdot \dfrac{7}{12x^4}$

____

5. $\dfrac{x^2 - 25}{x^2 - 16} \cdot \dfrac{x^2 - 4x}{2x + 10}$

____

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◆ Skill B Dividing rational expressions

Recall To divide, multiply by the reciprocal of the divisor.

◆ Example 1

Simplify. $\frac{x^2 - 9}{x^2 + x} \div \frac{x - 3}{x^2 - 1}$

◆ Solution

$$\frac{x^2 - 9}{x^2 + x} \div \frac{x - 3}{x^2 - 1} = \frac{x^2 - 9}{x^2 + x} \cdot \frac{x^2 - 1}{x - 3}$$

$$= \frac{(x + 3)\cancel{(x - 3)}\cancel{(x + 1)}(x - 1)}{x\cancel{(x + 1)}\cancel{(x - 3)}}$$

$$= \frac{(x + 3)(x - 1)}{x}$$

◆ Example 2

Simplify the complex fraction. $\dfrac{\frac{x^2 - 1}{x^2 + 3x + 2}}{\frac{x^2 - 2x + 1}{x + 2}}$

◆ Solution

$$\dfrac{\frac{x^2 - 1}{x^2 + 3x + 2}}{\frac{x^2 - 2x + 1}{x + 2}} = \frac{x^2 - 1}{x^2 + 3x + 2} \cdot \frac{x + 2}{x^2 - 2x + 1}$$

$$= \frac{\cancel{(x + 1)}\cancel{(x - 1)}\cancel{(x + 2)}}{\cancel{(x + 1)}\cancel{(x + 2)}\cancel{(x - 1)}(x - 1)}$$

$$= \frac{1}{x - 1}$$

Simplify each expression.

7. $\frac{3x^3}{5} \div \frac{6x^5}{8}$

____

9. $\frac{x^2}{x^2 + 2x + 1} \div \frac{3x}{x^2 - 1}$

____

11. $\dfrac{\frac{3x^2 - 9x}{x - 2}}{\frac{x^2 - 9}{4x - 8}}$

____

◆ Skill C Adding and subtracting rational expressions with unlike denominators

Recall To add or subtract two fractions, they must have a common denominator.

◆ **Example**

Simplify. $\frac{x+2}{x^2-9} - \frac{7}{4x+12}$

◆ **Solution**

Factor the denominators: $(x+3)(x-3)$ and $4(x+3)$.
Then find the LCM of $(x+3)(x-3)$ and $4(x+3)$. The LCM is $4(x+3)(x-3)$.
Multiply each fraction by 1 so that they will both have the same denominator.

$$\frac{x+2}{x^2-9} - \frac{7}{4x+12} = \frac{x+2}{(x+3)(x-3)} \cdot \frac{4}{4} - \frac{7}{4(x+3)} \cdot \frac{x-3}{x-3}$$

$$= \frac{(x+2)\cdot 4 - 7(x-3)}{4(x+3)(x-3)} \qquad \textit{where } \frac{4}{4} = 1 \textit{ and } \frac{x-3}{x-3} = 1$$

$$= \frac{4x + 8 - 7x + 21}{4(x+3)(x-3)}$$

$$= \frac{-3x + 29}{4(x+3)(x-3)}$$

Simplify.

10. $\frac{2}{7x} + \frac{11}{14x} - \frac{2}{21x}$ __________

11. $\frac{3y}{6y-12} - \frac{2y}{y-2}$ __________

12. $\frac{3}{x+6} + \frac{5}{2x-3}$ __________

13. $\frac{10}{a^2-2a} - \frac{5}{a^2-4}$ __________

14. $\frac{5}{x^2-3x-4} - \frac{3}{x^2-x-2}$ __________

15. $\frac{1}{x} - \frac{2}{x+2} - \frac{2}{x^2+3x+2}$ __________

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Reteaching
8.5 Solving Rational Equations and Inequalities

◆ **Skill A** Solving rational equations

Recall You should find the domain before solving the equation.

◆ **Example**

Solve. $\frac{x}{x-3} - \frac{7x-6}{x^2-x-6} = \frac{2}{x+2}$

◆ **Solution**

1. Factor $x^2 - x - 6 = (x-3)(x+2)$ so that you can determine the domain. The **domain** is all real numbers except 3 and -2.
2. The **common denominator** for all of the fractions is $(x-3)(x+2)$.
3. Multiply every term by the common denominator.
$$\frac{x}{x-3}(x-3)(x+2) - \frac{7x-6}{(x-3)(x+2)}(x-3)(x+2) = \frac{2}{x+2}(x-3)(x+2)$$
4. Simplify this equation so that no fractions remain.
$$x(x+2) - (7x-6) = 2(x-3)$$
$$x^2 + 2x - 7x + 6 = 2x - 6$$
$$x^2 - 7x + 12 = 0$$
$$(x-3)(x-4) = 0$$
$$x = 3 \quad \text{or} \quad x = 4$$
But $x = 3$ is not in the domain, so the solution is $x = 4$.

Check by making a table of values for $y_1 = \frac{x}{x-3} - \frac{7x-6}{x^2-x-6}$ and $y_2 = \frac{2}{x+2}$.
Identify where $y_1 = y_2$.
The corresponding x-value is a solution.

X	Y1	Y2
0	-1	1
1	-.3333	.66667
2	0	.5
3	ERROR	.4
4	.33333	.33333
5	.42857	.28571
6	.5	.25
X=0		

Solve each equation. Check your solution.

1. $\frac{1}{4x} - \frac{3}{4} = \frac{7}{x}$ ______________________
2. $\frac{2}{x+4} + \frac{3}{x+4} = 10$ ______________________
3. $\frac{5}{2x-2} = \frac{15}{x^2-1}$ ______________________

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8.7 Simplifying Radical Expressions

◆ Skill A Simplifying radicals

Recall If n is an even integer, then $\sqrt[n]{a^n} = |a|$; for example, $\sqrt[4]{(-2)^4} = |-2| = 2$.
If n is an odd integer, then $\sqrt[n]{a^n} = a$; for example, $\sqrt[3]{(-2)^3} = -2$.

◆ Example
Simplify $\sqrt{50a^3bc^4}$.

◆ Solution

$\sqrt{50a^3bc^4} = \sqrt{25a^2c^4 \cdot 2ab}$ *Make one monomial a perfect square.*
$= \sqrt{25a^2c^4} \cdot \sqrt{2ab}$ *Product Property of Radicals*
$= 5|a|c^2\sqrt{2ab}$ *c^2 is always nonnegative. Thus, absolute value is not needed.*

Simplify each radical expression by using the Properties of *n*th Roots.

1. $\sqrt{48x^3y^4}$ ________ **2.** $\sqrt{8x^6y^5}$ ________ **3.** $\sqrt{150m^{12}}$ ________

4. $\sqrt{175a^2b^3c^4}$ ________ **5.** $\sqrt[3]{54x^6}$ ________ **6.** $\sqrt[5]{32r^{12}x^{10}}$ ________

◆ Skill B Simplifying products and quotients of radical expressions

Recall Product Property of Radicals: $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
Quotient Property of Radicals: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ where $b \neq 0$

◆ Example
Simplify each expression. Assume that the value of each variable is positive.

a. $(18xy^2)^{\frac{1}{2}} \cdot \sqrt{2xz^3}$ **b.** $\frac{(24x^3y^7)^{\frac{1}{3}}}{\sqrt[3]{3y^2}}$

◆ Solution

a. $(18xy^2)^{\frac{1}{2}} \cdot \sqrt{2xz^3} = \sqrt{18xy^2 \cdot 2xz^3}$ *Product Property of Radicals*
$= \sqrt{36x^2y^2z^2 \cdot z}$
$= \sqrt{36x^2y^2z^2} \cdot \sqrt{z}$
$= 6xyz\sqrt{z}$

No absolute value is necessary, since values of variables are assumed to be positive.

b. $\frac{(24x^3y^7)^{\frac{1}{3}}}{\sqrt[3]{3y^2}} = \sqrt[3]{\frac{24x^3y^7}{3y^2}}$
$= \sqrt[3]{8x^3y^5} = \sqrt[3]{8x^3y^3 \cdot y^2} = \sqrt[3]{8x^3y^3} \cdot \sqrt[3]{y^2} = 2xy\sqrt[3]{y^2}$

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Simplify each product or quotient. Assume that the value of each variable is positive.

7. $\sqrt{6x^2} \cdot \sqrt{6x^2}$ ________

8. $\sqrt{3x^4} \cdot \sqrt{12x^3}$ ________

9. $(15x^4y^2)^{\frac{1}{2}} \cdot \sqrt{5y^6}$ ________

10. $\dfrac{\sqrt{54x^4y^6}}{\sqrt{6xy^4}}$ ________

11. $\dfrac{(216a^9)^{\frac{1}{3}}}{\sqrt[3]{a^6}}$ ________

12. $\dfrac{\sqrt[3]{108a^{15}b^{10}}}{(2b)^{\frac{1}{3}}}$ ________

◆ Skill C Performing operations on radical expressions

Recall By using "FOIL," $(a + \sqrt{b})(c + \sqrt{d}) = ac + a\sqrt{d} + c\sqrt{b} + \sqrt{bd}$.

◆ **Example 1**

Simplify by performing the indicated operations: $(2 + \sqrt{8})(5 - \sqrt{2}) + 3\sqrt{2}$.

◆ **Solution**

$$\begin{aligned}(2 + \sqrt{8})(5 - \sqrt{2}) + 3\sqrt{2} &= 10 - 2\sqrt{2} + 5\sqrt{8} - \sqrt{16} + 3\sqrt{2} \\ &= 10 - 2\sqrt{2} + 10\sqrt{2} - 4 + 3\sqrt{2} \\ &= 10 - 4 + \sqrt{2}(-2 + 10 + 3) \\ &= 6 + 11\sqrt{2}\end{aligned}$$

◆ **Example 2**

Rationalize the denominator of each expression. **a.** $\dfrac{8}{\sqrt{2}}$ **b.** $\dfrac{6}{\sqrt{3} + 2}$

◆ **Solution**

a. $\dfrac{8}{\sqrt{2}} = \dfrac{8}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$ *Multiply by* $\dfrac{\sqrt{2}}{\sqrt{2}}$.

$= \dfrac{8\sqrt{2}}{2}$

$= 4\sqrt{2}$

b. $\dfrac{6}{\sqrt{3} + 2} = \dfrac{6}{\sqrt{3} + 2} \cdot \dfrac{\sqrt{3} - 2}{\sqrt{3} - 2}$

$= \dfrac{6\sqrt{3} - 12}{(\sqrt{3})^2 - (2)^2}$

$= \dfrac{6\sqrt{3} - 12}{-1}$

Find each sum, difference, or product. Give your answer in simplest radical form. Assume that the value of each variable is positive.

13. $(\sqrt{11} - \sqrt{13})(\sqrt{11} + \sqrt{13})$ ________

14. $(2\sqrt{x} + 1)(\sqrt{x} - 4)$ ________

15. $3\sqrt{7}(\sqrt{7} - \sqrt{14})$ ________

16. $(3\sqrt{y} - 1)(2\sqrt{y} + 4) - 10\sqrt{y}$ ________

Write each expression with a rational denominator and in simplest form.

17. $\dfrac{3}{\sqrt{2}}$ ________

18. $\dfrac{3\sqrt{7}}{7\sqrt{3}}$ ________

19. $\dfrac{4}{3 - 2\sqrt{2}}$ ________